SET THEORY HOMEWORK 5

Due Thursday, April 20.

Problem 1. *Suppose that* $\mathbb{P}$ *is a poset,* $A \subset \mathbb{P}$ *is a maximal antichain,* $\phi(x)$ *is a formula, and* $\langle \tau_p \mid p \in A \rangle$ *are* $\mathbb{P}$ *names such that for all* $p \in A$, $p \Vdash \phi(\tau_p)$. *Then there is a* $\mathbb{P}$ *name* τ, *such that* $1_{\mathbb{P}} \Vdash \phi(\tau)$.

We say that σ is a *nice name* for a subset of ω, if for all $n \in \mathrm{dom}(\sigma)$, $\{q \mid \langle n, q \rangle \in \sigma\}$ is an antichain.

Problem 2. *Suppose that* $M \models \kappa^\omega = \kappa$, *and let* G *be* $Add(\omega, \kappa)$*-generic over* M. *Then* $M[G] \models 2^\omega = \kappa$.

Hint: we already showed that $M[G] \models 2^\omega \geq \kappa$. *For the other direction, argue that it is enough to consider the possible number of nice names for subsets of* ω. *I.e. for any* $a \subset \omega$ *in* $M[G]$, *show there is a nice name* $\dot{a}$ *such that* $\dot{a}_G = a$. *Then use the c.c.c. to compute the number of nice names.*

Problem 3. *Suppose that* $\mathbb{P}, \mathbb{Q}$ *are two posets in the ground model* V *and* $\pi : \mathbb{P} \to \mathbb{Q}$ *is a projection, i.e.*

(1) π *is order preserving:* $p' \leq p \to \pi(p') \leq \pi(p)$,
(2) *for all* $p \in \mathbb{P}$ *and* $q \leq \pi(p)$, *there is* $p' \leq p$, *such that* $\pi(p') \leq q$.

Suppose that G *is a* $\mathbb{P}$*-generic filer over* V. *Show that* $H := \{q \mid \exists p \in G, \pi(p) \leq q\}$ *is a* $\mathbb{Q}$*-generic filter over* V.

Problem 4. *Let* $\mathbb{P}$ *be a poset such that for any condition* p, *there are incompatible* $q, r \leq p$. *Suppose that* G *is* $\mathbb{P}$*-generic. Show that* $G \times G$ *is not* $\mathbb{P} \times \mathbb{P}$*-generic.*

Problem 5. *Suppose that for all* n, $2^{\aleph_n} = \aleph_{\omega+1}$. *Show that* $2^{\aleph_\omega} = \aleph_{\omega+1}$. *Hint: For each* $A \subset \aleph_\omega$, *define* $A_n := A \cap \aleph_n$. *Consider the map* $A \mapsto \langle A_n \mid n < \omega \rangle$.